Name:___________________________

SM3 7.3: Graphing Tangent & Cotangent

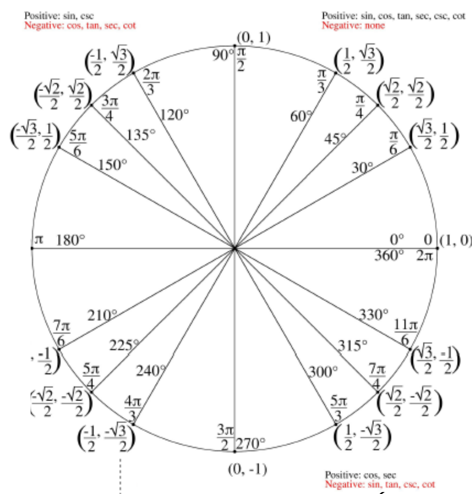
__Vocabulary:__ period, asymptote

__Tangent Notes:__ Recall that $\tan x = \frac{\sin x}{\cos x}$. Let's compare several values to determine some characteristics of the tangent function.

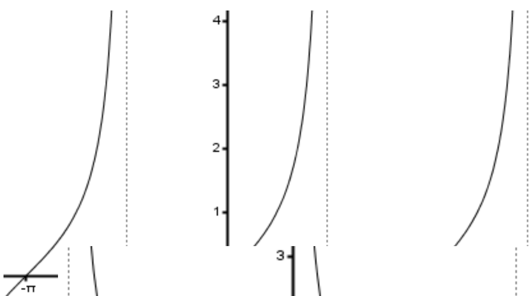
x	$\sin x$	$\cos x$	$\tan x = \frac{\sin x}{\cos x}$
0	0	1	$\frac{0}{1} = 0$
$\frac{\pi}{2}$	1	0	$\frac{1}{0} = Undefined$
π	0	-1	$\frac{0}{-1} = 0$
$\frac{3\pi}{2}$	-1	0	$-\frac{1}{0} = Undefined$
2π	0	1	$\frac{0}{1} = 0$

The tangent function has zeros everywhere that $\sin x = 0$. The tangent function has asymptotes whenever the $\cos x = 0$ because this makes the tangent function undefined.

If you consider the tangent values on the unit circle, you will notice that they repeat every half turn of the circle (instead of every full turn like sine and cosine), making the period of tangent π.



Below is a graph of the tangent function.

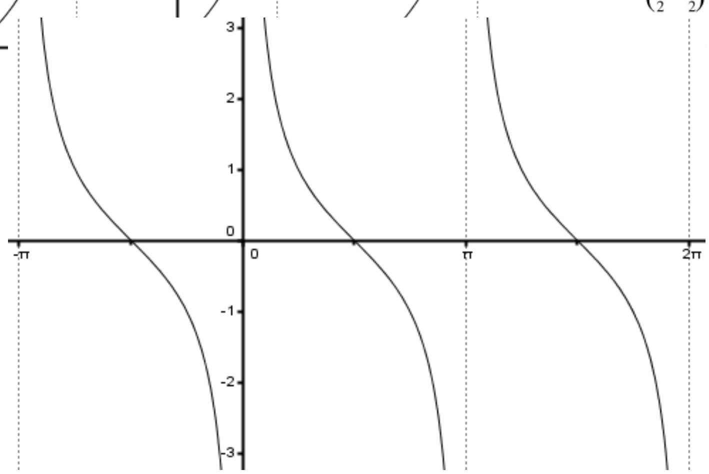


Extended form of the tangent function:

$$y = a\tan\big(b(x-h)\big) + k$$

Period of the tangent function: $\frac{\pi}{b}$

__Cotangent Notes:__ Since the cotangent is the reciprocal of the tangent function ($\cot x = \frac{\cos x}{\sin x}$), then it has asymptotes where $\sin x = 0$ and zeros where $\cos x = 0$. It also has the same period as tangent, π. The graph of $y = \cot x$ is shown to the right.



Extended form: $y = a\cot\big(b(x-h)\big) + k$

Period: $\frac{\pi}{b}$

NAME________________________

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Identify the period and first asymptote on the positive side of the x-axis.

1) $y = 2\tan x$

2) $f(x) = 3\cot(x)$

3) $f(x) = \tan\left(\frac{1}{2}x\right)$

4) $y = \cot(2x)$

5) $y = \tan(x) - 3$

6) $h(x) = 1 - 4\cot(x)$

7) $g(x) = \tan\left(x - \frac{\pi}{3}\right)$

8) $f(x) = \cot 4\left(x + \frac{\pi}{8}\right)$

Describe how changes in the given variable change the shape of the tangent curve:

$$y = a\tan\big(b(x-h)\big) + k$$

9) $k = 2$

10) $k = -\frac{1}{3}$

11) $a = 2$

12) $a = \frac{1}{3}$

13) $b = 2$

14) $b = \frac{1}{3}$

15) $h = \pi$

16) $h = -\frac{\pi}{4}$

Sketch an appropriate coordinate axis and graph two periods of the function.

17) $y = 3\tan x$

Per:	
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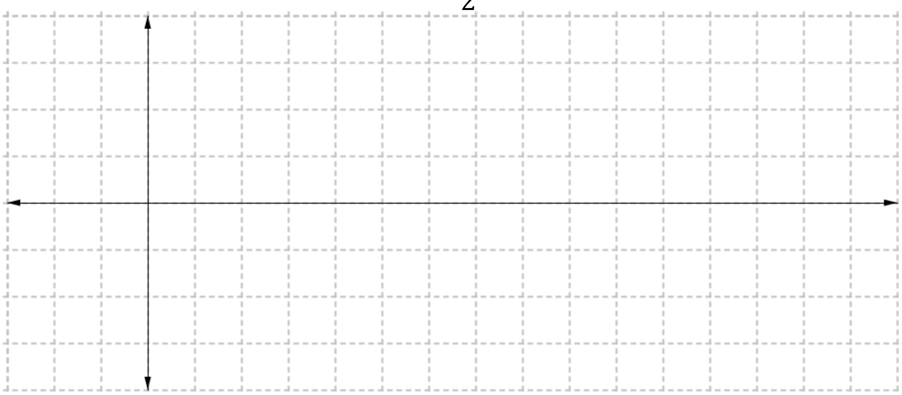
18) $f(x) = \frac{1}{2}\cot x$



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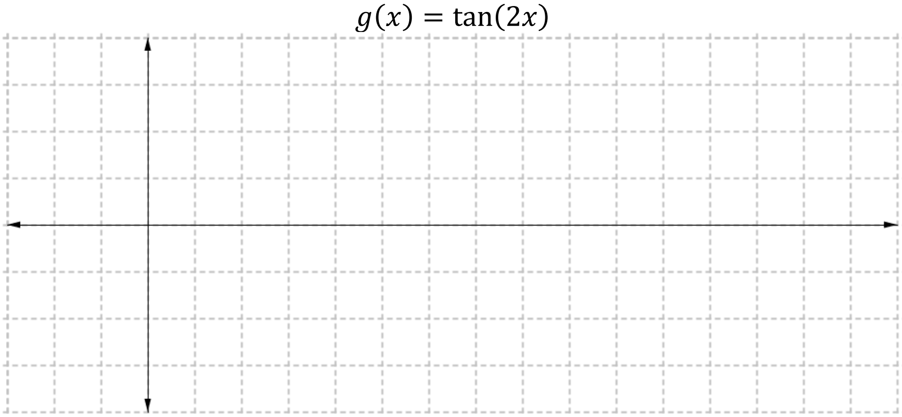
19) $g(x) = \tan(2x)$



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20) $y = \cot\left(\frac{x}{2}\right)$

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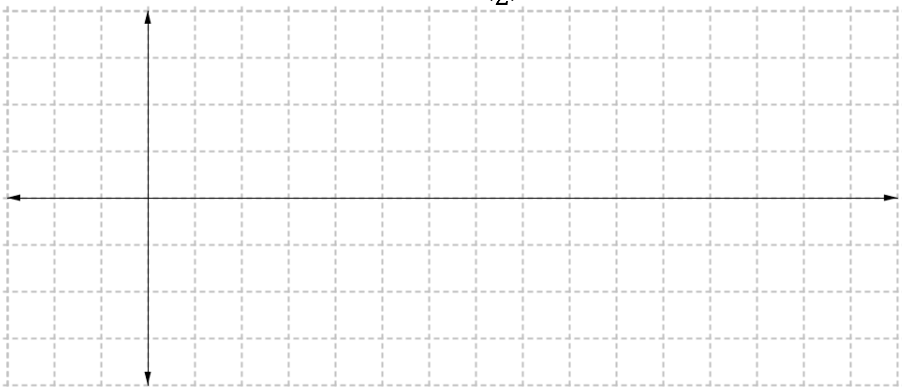

21) $y = \tan(x + \pi)$

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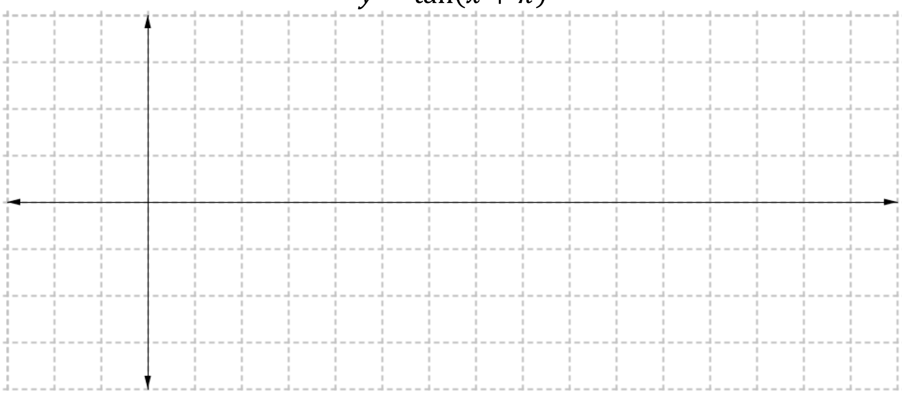

22) $f(x) = 2\cot x + 2$

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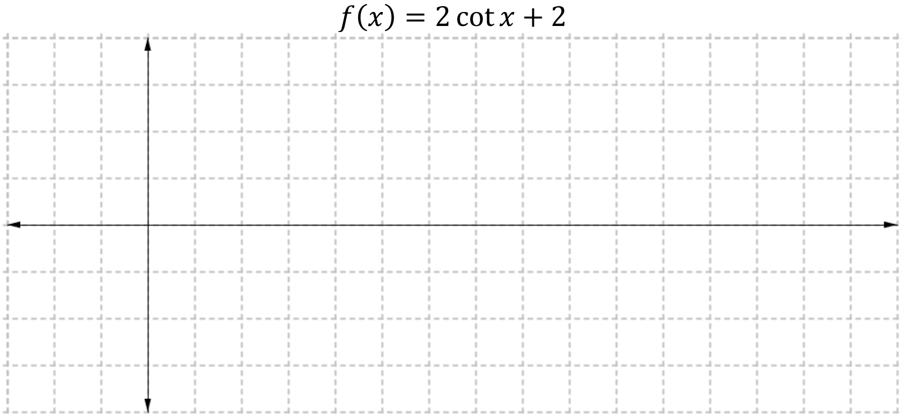
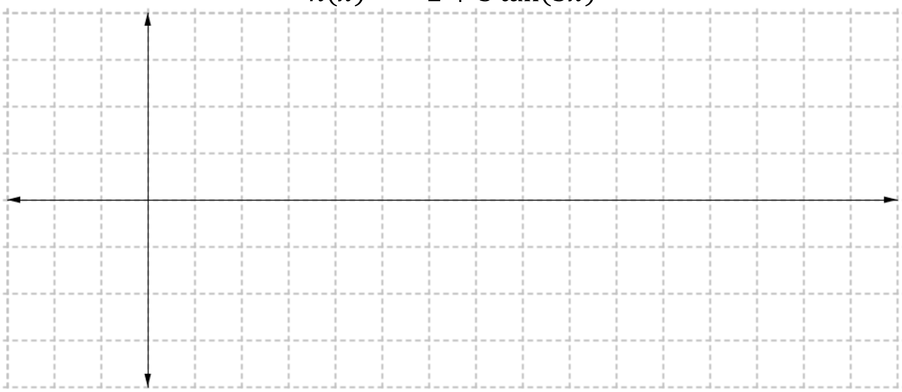


23) $h(x) = -2 + 3\tan(3x)$



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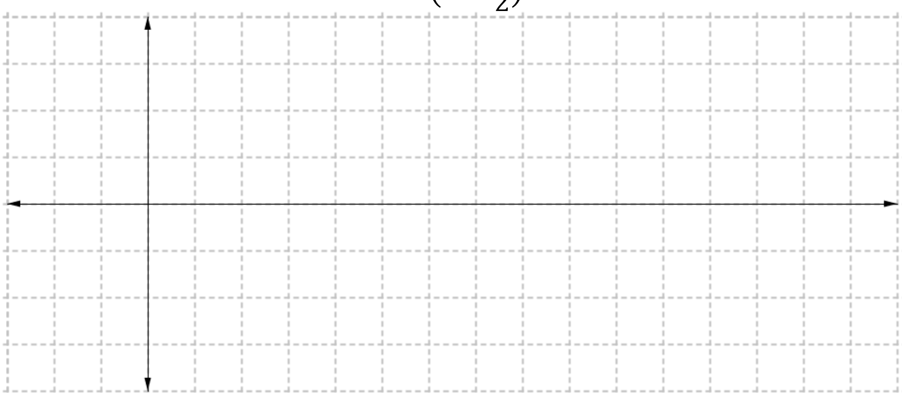
24) $y = \cot\left(x - \frac{\pi}{2}\right) - 1$



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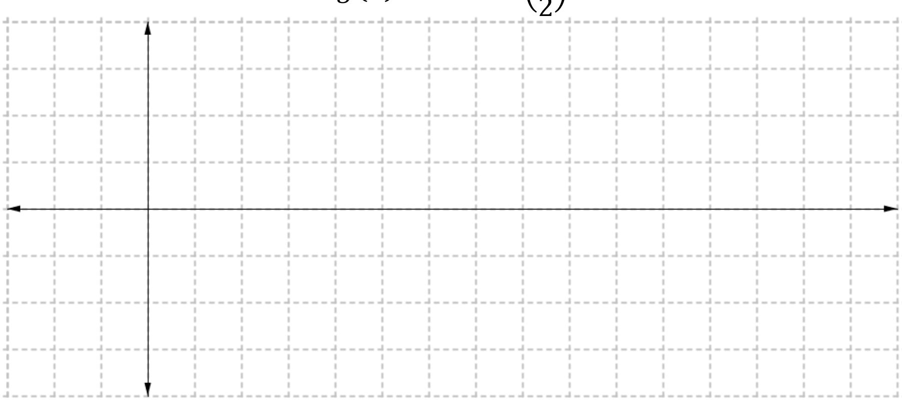
25) $g(x) = -4\tan\left(\frac{x}{2}\right)$



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26) $y = -\cot(5x)$

Per:	
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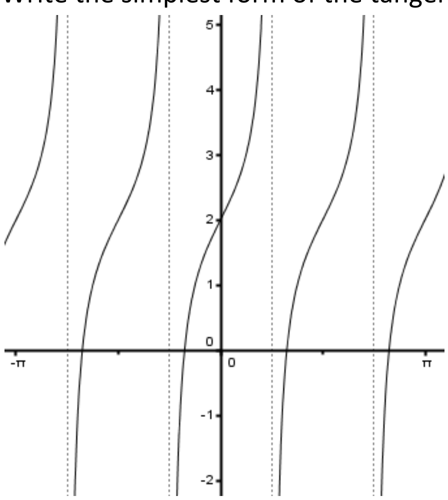
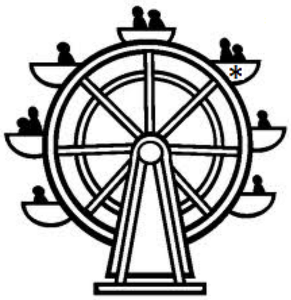
27) Write the simplest form of the tangent function for the given graph.



Review and application: Answer the questions about the situation:



A ferris wheel with 8 carriages has a radius of 40 feet. The wheel spins at a rate of 2.5 revolutions per minute. The bottom of the wheel has a clearance of 8 feet above the ground.

Monica and Shelly climb into a carriage at the bottom position using a retractable stair case.

28) Fill out the time and height values in the table, indicating how far the girls are from the ground t seconds after the ferris wheel begins moving (use t values that correspond to the carriage positions shown in the graphic):

Time (t):	0									
Height:										

29) What is the amplitude of the wave that represents the height of a carriage on the ferris wheel (include units)?

30) As the wheel moves, it follows the points of a trigonometric curve. Which trigonometric function describes the height of a carriage without needing a phase shift: $\sin x$, $-\sin x$, $\cos x$, or $-\cos x$?

31) Use the formula $Period = \frac{2\pi}{b}$ to determine an appropriate value for (b):

32) How far is the wheel from the ground?

33) Write a trigonometric function that accurately describes the height of the wheel as a function of time: